NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

AN ANALYSIS OF AN AUTO-ALERT SONOBUOY DETECTION MODEL

by

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March 1988

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REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY	3 DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; Distribution is unlimited
2b DECLASSIFICATION/DOWNGRADING SCHEDULE	
4 PERFORMING ORGANIZATION REPORT NUMBER(S)	5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a NAME OF PERFORMING ORGANIZATION Naval Postgraduate School 6b OFFICE SYMBOL (If applicable) 55	7a NAME OF MONITORING ORGANIZATION Naval Postgraduate School
6c ADDRESS (City, State, and ZIP Code) Monterey, California 93943-5000	7b ADDRESS (City, State, and ZIP Code) Monterey, California 93943-5000
8a NAME OF FUNDING/SPONSORING ORGANIZATION 8b OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c ADDRESS (City, State, and ZIP Code)	10 SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO · PROJECT NO · TASK NO · WORK UNIT ACCESSION NO

11 TITLE (Include Security Classification) AN ANALYSIS OF AN AUTO-ALERT SONOBUOY DECTECTION MODEL

12 PERSONAL AUTHOR(S) Filanowicz, Robert Walter

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
Master's Thesis	FROM ______ TO ______	1988 MARCH	65

16 SUPPLEMENTARY NOTATION The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government

17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Auto-Alert, Sonobuoy

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

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20 DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21 ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a NAME OF RESPONSIBLE INDIVIDUAL R. N. Forrest	22b TELEPHONE (Include Area Code) (408) 646-2653 · 22c OFFICE SYMBOL 55Fo

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete

☆ U.S. Government Printing Office: 1986—606-24.

An Analysis of an Auto-Alert Sonobuoy Detection Model

by

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
March 1988

ABSTRACT

This thesis presents a method for estimating some parameter values that are involved in the design of a passive broadband auto-alert sonobuoy. The importance of establishing a false alarm rate that is compatible with tactical requirements is stressed. A FORTRAN program is listed for estimating an auto-alert sonobuoy lateral range curve.

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I. INTRODUCTION

The passive SONAR equation and a detection model can be combined to provide a tool for making preliminary estimates of performance in the design of a passive broadband auto-alert sonobuoy. (A review of the passive SONAR equation is provided in Appendix A.) In Chapter II, a computer simulation is described that illustrates this. In this case, the detection model is based on a Square Law detector. In Chapter III, some of the tactical considerations inherent in the establishment of an acceptable false alarm rate are discussed. The importance of matching integration time to signal duration is illustrated in conjunction with a method for estimating a tactically optimum integration time. In addition, auto-alert design parameter values are compared under differing decision criteria (WALD, LAPLACE, and MINIMUM REGRET) as a method for determining an optimum design. The original intent of this thesis was to estimate an auto-alert sensor's performance based on the proposed design parameters and then to compare the estimates to operational data. No suitable data was available so hypothetical systems and parameters were used. The analysis indicates that an auto-alert sensor's performance can be adequately described by a Definite Range Law ("Cookie Cutter") model. Appendices D and E provide a listing of the Fortran code and key variables used in the computer simulation detection model.

II. COMPUTER MODEL

The statistical basis for adopting a detection model based on a Square Law detector is developed in Appendix B. It is shown there that a Square Law detector is an optimum detector if the stochastic processes that determine both the signal and the noise are stationary gaussian processes. The Square Law detector's test statistic is $X = \Sigma Y_i^2$ where Y_i is a random input value and i is a time index. Figure 2.1 is a block diagram of a Square Law detector.

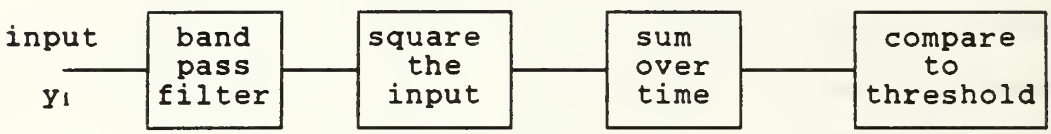

Figure 2.1: Square Law Detector

The computer algorithm simulates a Square Law detector for a case in which the signal process is not stationary. It is a time step simulation in which independent noise and signal random variables are generated at each time step. Their sum is then squared and the total is accumulated over an integration period. The total is then compared to a threshold value to determine if a detection has occurred.

A. ASSUMPTIONS

In particular, the computer simulation model is based on the following assumptions:

1. The input to a hydrophone is determined by the sum of two independent gaussian stochastic processes: a stationary noise process and a non-stationary signal process.

2. The random variables determined by the processes are independent and their means are both equal to zero. The variance of the noise random variable is σ^2 and the variance of the signal random variable is $\sigma_2(r)$ where r is the range of the target from the hydrophone.

3. σ^2 is the noise and $\sigma^2(r)$ is the signal in the passive SONAR equation.

4. Transmission loss is due to spherical spreading (attenuation due to absorption is negligible).

5. All encounters are straight line encounters.

The model implies that $S(r)$ may be written as

$$10 \log[S(r)] = SL - TL(r) \qquad \text{(eqn 2.1)}$$

where:

$$TL(r) = 20 \log(r) \quad \text{and,} \qquad \text{(eqn 2.2)}$$

r is the range from the target in yards. A specified false alarm probability p_f determines a threshold value for the statistic X. The procedure is an analytical one that is the same as that described in Appendix B. However, unlike the case in Appendix B, the distribution of X can not be easily determined analytically. The purpose of the computer simulation is to determine values for the probability of detection during an encounter and with these values to establish a lateral range curve for a Square Law detector when the signal stochastic process is not stationary.

B. METHODOLOGY FOR CALCULATING P_d

Probability of detection during an encounter is estimated empirically by $\hat{p}_d$ where

$$\hat{p}_d = \frac{\text{number of detections}}{\text{repetitions}} . \qquad \text{(eqn 2.3)}$$

A detection occurs when the value of the test statistic for an integration period exceeds the preset threshold. One

hundred repetitions were used. Ideally, a larger number of repetitions is desired in order to achieve a more statistically valid lateral range curve but a compromise was made in order to limit computer run time. Pseudonormal random numbers for the simulation were generated using the LLRANDOMII package as installed on the IBM 3033 at the Naval Postgraduate School [Ref. 1].

The simulation procedure requires approximately 45 minutes of processing time on an IBM 3033 computer to obtain data for three range values. Batch processing limitations necessitated running the program in increments with three data points on each run. Rather than submitting the input lateral range values as groups of data in ascending or descending order, the order was randomized. Even though the LLRANDOMII package is considered a reliable random number generator, this procedure was initiated to limit the possiblity of serial correlation between the random number streams and the range values. If any correlation did occur, it should be in the form of a random error rather than a systematic error [Ref. 2:p. 102]. To reduce the number of runs, reasonable lateral range values for the simulation were obtained by using the Square Law model of Appendix B for selected values of p_d.

C. SIMULATION RESULTS

Simulation runs were made with the same environmental and target parameters discussed in the examples of Chapter III. P_f was initially fixed at 10^{-4} and integration time was varied. The first four runs used integration times equal to 180, 300, 365, and 390 seconds. The 365 second run was

chosen to coincide with the tactical scenario of Chapter III. The other times were chosen to explore a spread of integration times to see if a trend could be established when comparing the lateral range curves generated by the Square Law simulation model with those calculated using the Square Law model delineated in Appendix B. In the figures that follow, results obtained by using the Square Law computer simulation model are labeled "Simulation" while those that were obtained using the Square Law model of Appendix B are labeled "Square Law".

Figures 2.2 thru 2.5 provide a graphical comparison between the lateral range curves generated by each model. The simulation model data points were smoothed using fourth and fifth degree polynomials in conjunction with a least squares criterion. Square Law curves were smoothed using cubic spline interpolation. The data is summarized in Tables 2.1 to 2.4. Based on the data from these four cases a relationship between the simulation and the Square Law data was established. The linear model of Figure 2.6 which was constructed using a least squares fit displays this relationship. This linear model was used to estimate an MDR for the simulation model based on a constant signal Square Law calculation for MDR (i.e., an estimate of lateral range when $p_d = .5$). Two partial data runs (three points) were then made for T=240 and T=333 seconds with the data points clustered about the predicted MDR. With these two additional data points a "new" model was generated for estimating the simulation model MDR when the Square Law value was known.

T=180 seconds $p_f = 10^{-4}$

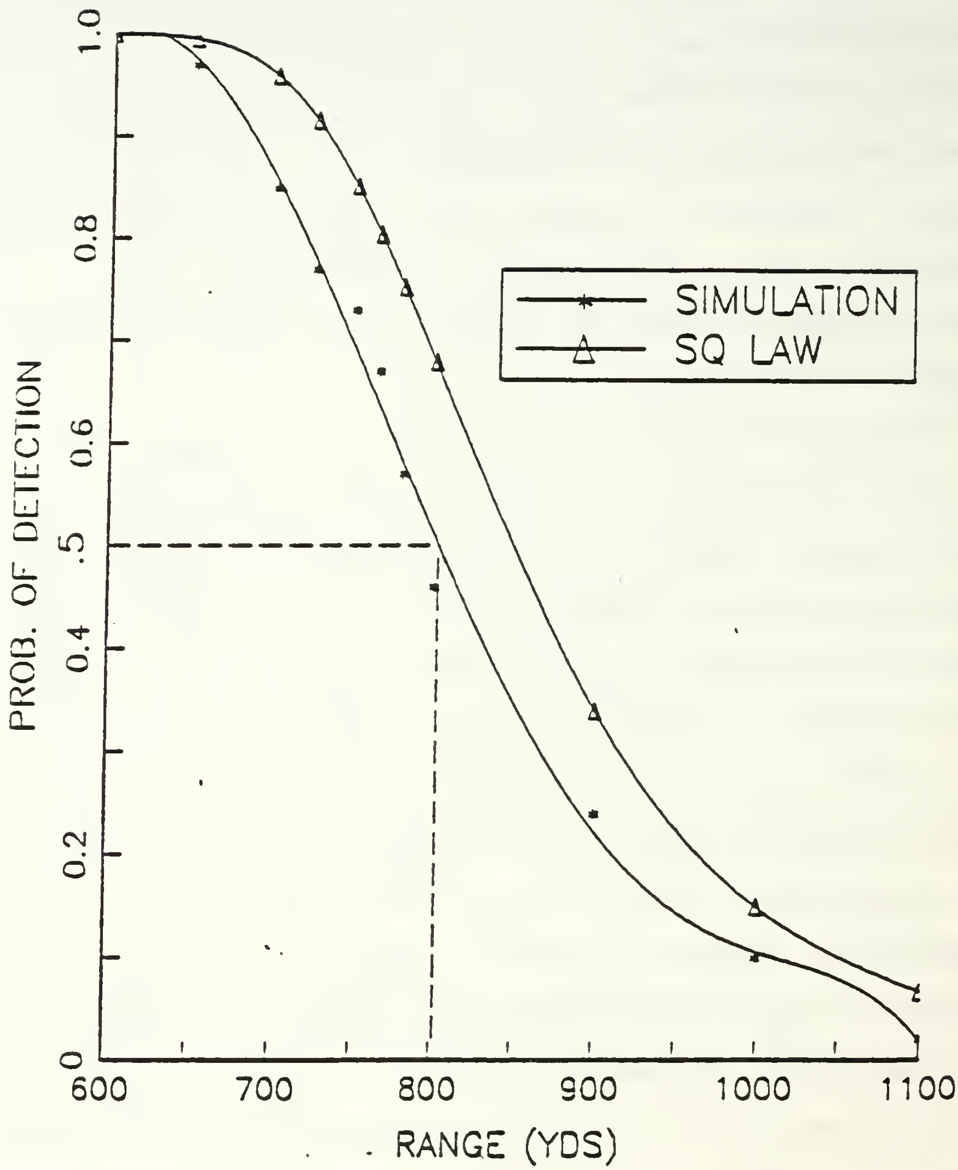

Figure 2.2: Lateral Range Curve
T=180 Seconds $P_f = 10^{-4}$

TABLE 2.1: LATERAL RANGE CURVE DATA
T=180 SECONDS $P_f = 10^{-4}$

LATERAL RANGE (YDS)	PROBABILITY OF DETECTION	
	SIMULATION	SQUARE LAW
600	1.00	.9997
650	.97	.9956
700	.85	.9595
725	.77	.9153
750	.73	.8509
765	.67	.8039
780	.57	.7522
800	.46	.6787
900	.24	.3395
1000	.10	.1488
1100	.02	.0660

T=300 seconds $p_f = 10^{-4}$

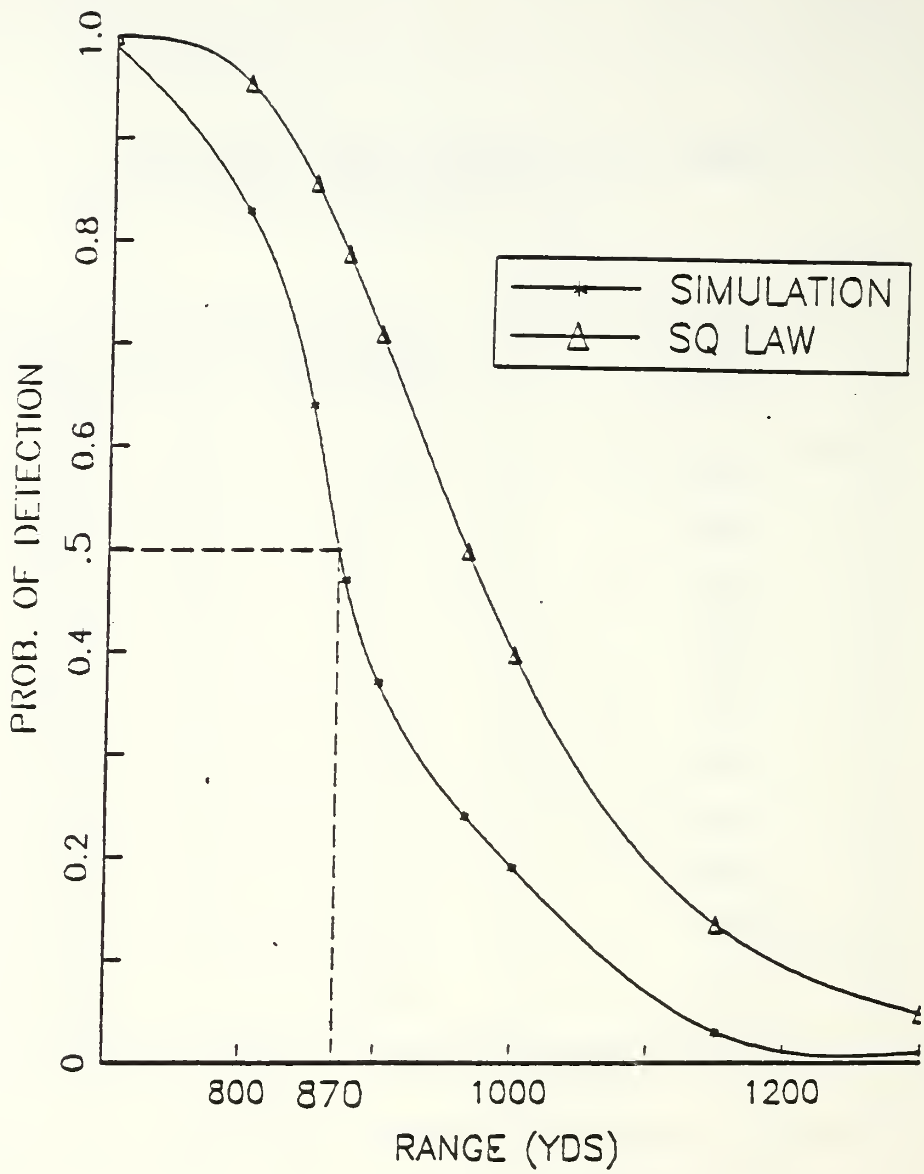

Figure 2.3: Lateral Range Curve
T=300 Seconds $P_f = 10^{-4}$

TABLE 2.2: LATERAL RANGE CURVE DATA
T=300 SECONDS $P_f = 10^{-4}$

LATERAL RANGE (YDS)	PROBABILITY OF DETECTION	
	SIMULATION	SQUARE LAW
700	.99	.9996
800	.83	.9536
850	.64	.8565
875	.47	.7868
900	.37	.7081
965	.24	.4970
1000	.19	.3964
1150	.03	.1345
1300	.01	.0471

T=365 $p_f = 10^{-4}$

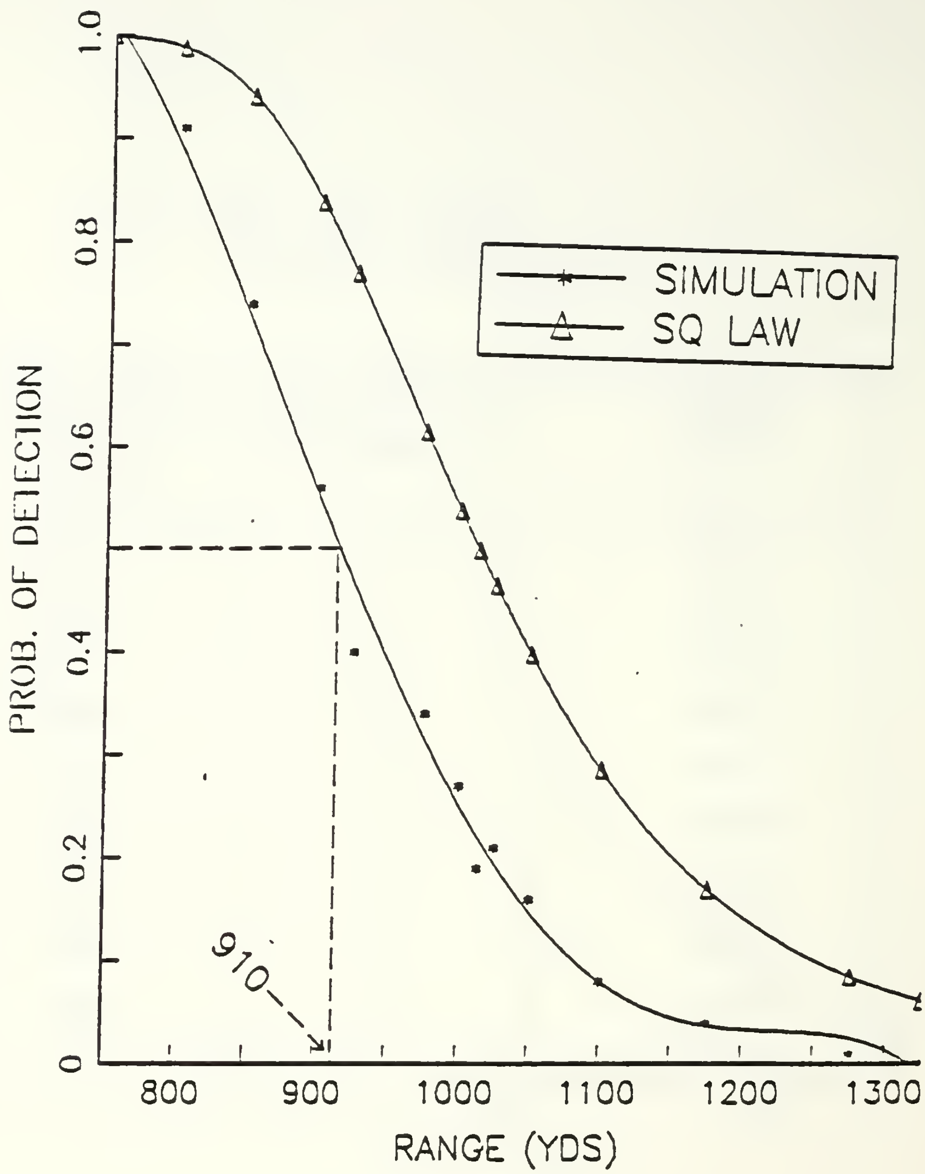

Figure 2.4: Lateral Range Curve
T=365 Seconds $P_f = 10^{-4}$

TABLE 2.3: LATERAL RANGE CURVE DATA
T=365 SECONDS $P_f = 10^{-4}$

LATERAL RANGE (YDS)	PROBABILITY OF DETECTION	
	SIMULATION	SQUARE LAW
750	1.00	.9989
800	.91	.9874
850	.74	.9463
900	.56	.8383
925	.40	.7693
975	.34	.6146
1000	.27	.5372
1013	.19	.4985
1025	.21	.4640
1050	.16	.3970
1100	.08	.2850
1175	.04	.1691
1275	.01	.0848
1325	.00	.0609

T=390 seconds p_f =10^{-4}

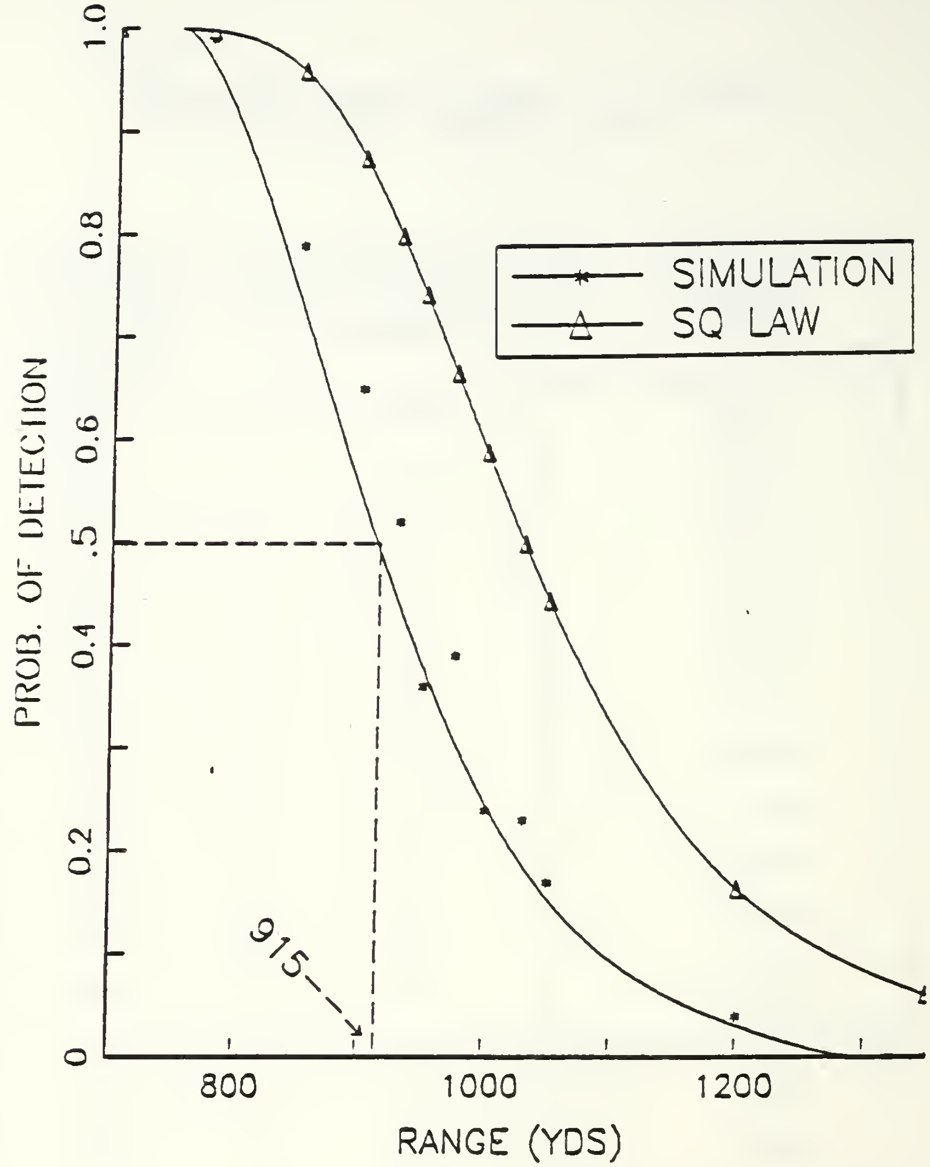

Figure 2.5: Lateral Range Curve
T=390 Seconds P_f =10^{-4}

TABLE 2.4: LATERAL RANGE CURVE DATA
T=390 SECONDS $P_f = 10^{-4}$

LATERAL RANGE (YDS)	PROBABILITY OF DETECTION	
	SIMULATION	SQUARE LAW
700	1.00	1.0000
775	.99	.9978
850	.79	.9586
900	.65	.8741
930	.52	.7988
950	.36	.7413
975	.39	.6651
1000	.24	.5877
1050	.17	.4425
1200	.04	.1630
1350	.00	.0598

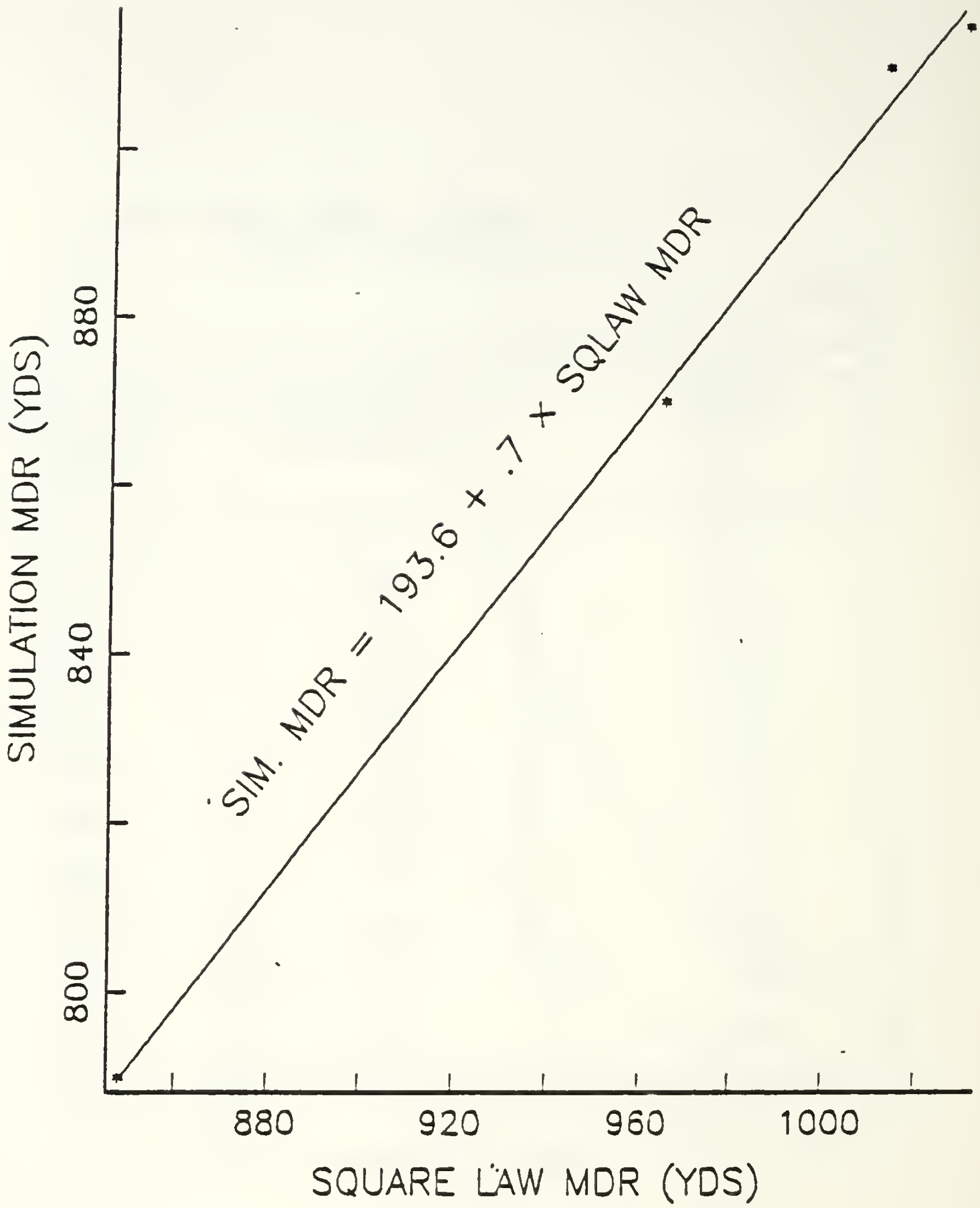

Figure 2.6: First Iteration of the Linear Model for Predicting Simulation MDR

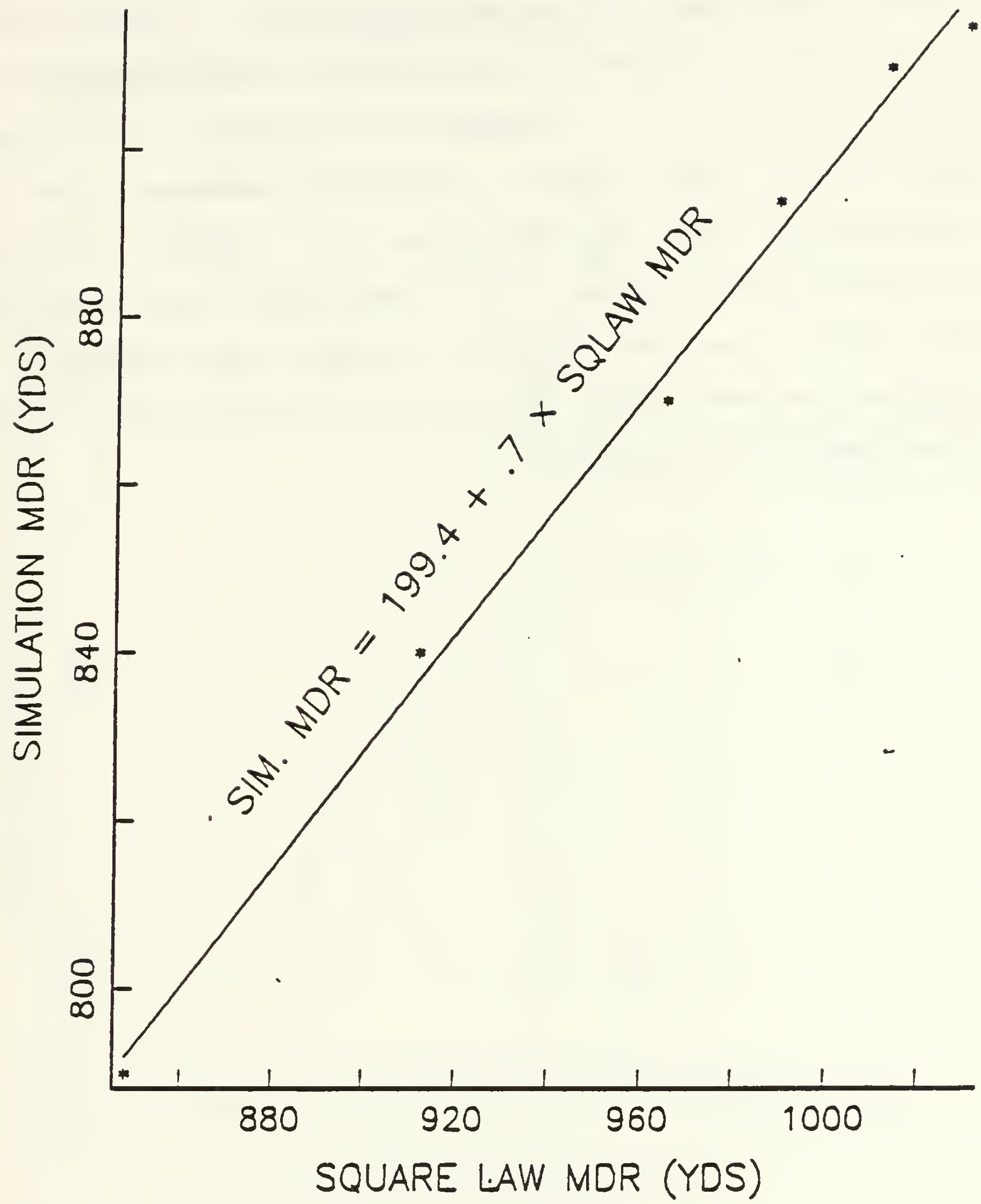

Figure 2.7: Second Iteration of the Linear Model for Predicting Simulation MDR

Data runs were also conducted with p_f as a variable. P_f values of 10^{-5} and 10^{-6} were used to investigate the "cost" in MDR associated with a decrease in p_f. When p_f decreased by a power of 10, MDR decreased by approximately 60 yards. Tables 2.5 thru 2.7 summarize the data. The integration times of 312, 333, and 365 seconds represent the "optimum" integration times as discussed in Chapter III for the particular value of p_f. An additional run with T=333 and $p_f=10^{-4}$ was done to provide a common time reference for two different values of p_f. Plotting the $\log_{10}$ of p_f yielded the linear model of Figure 2.8.

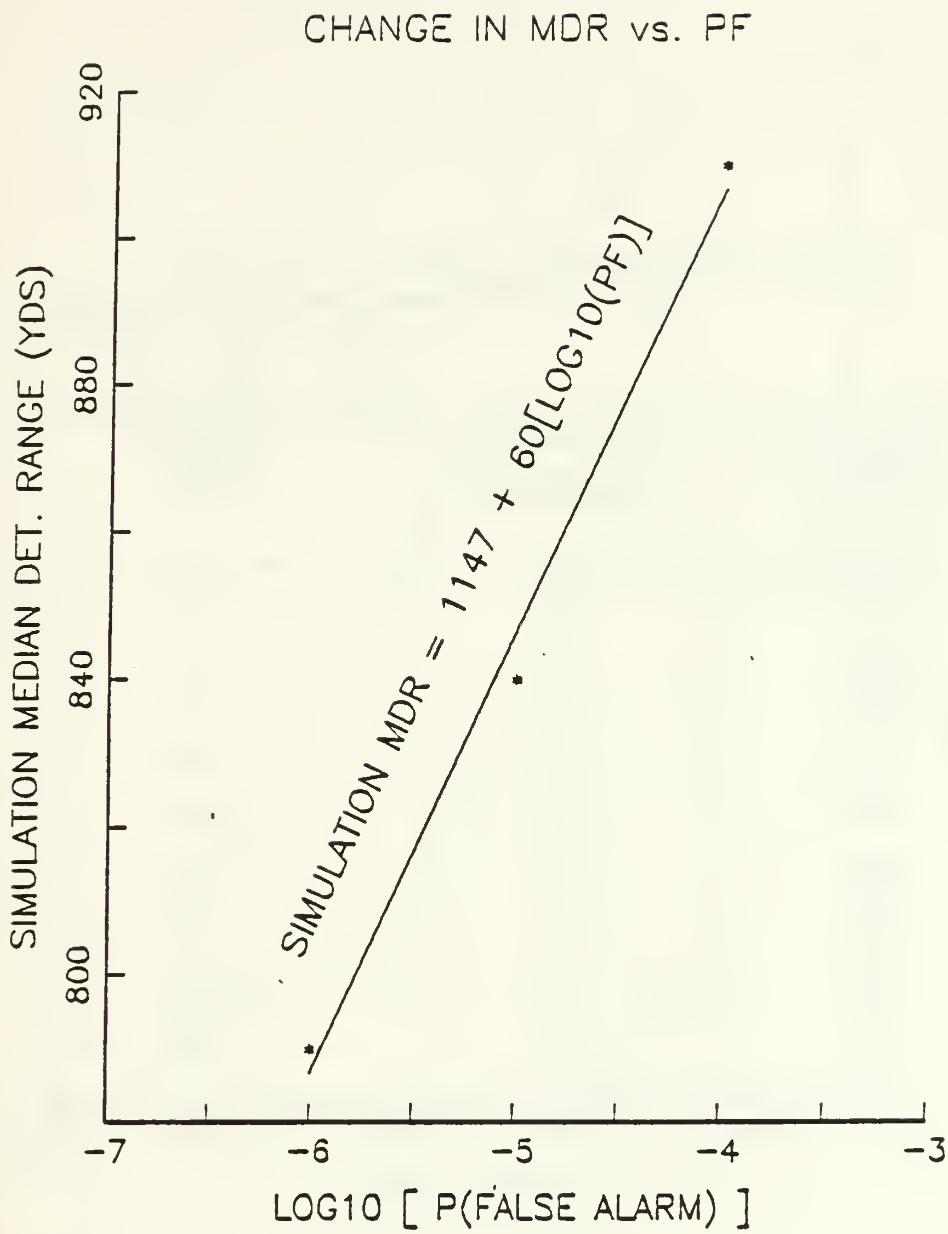

Figure 2.8: Change in Simulation MDR vs. an Order of Magnitude Change in P_f

T=333 seconds p_f=10^{-4}

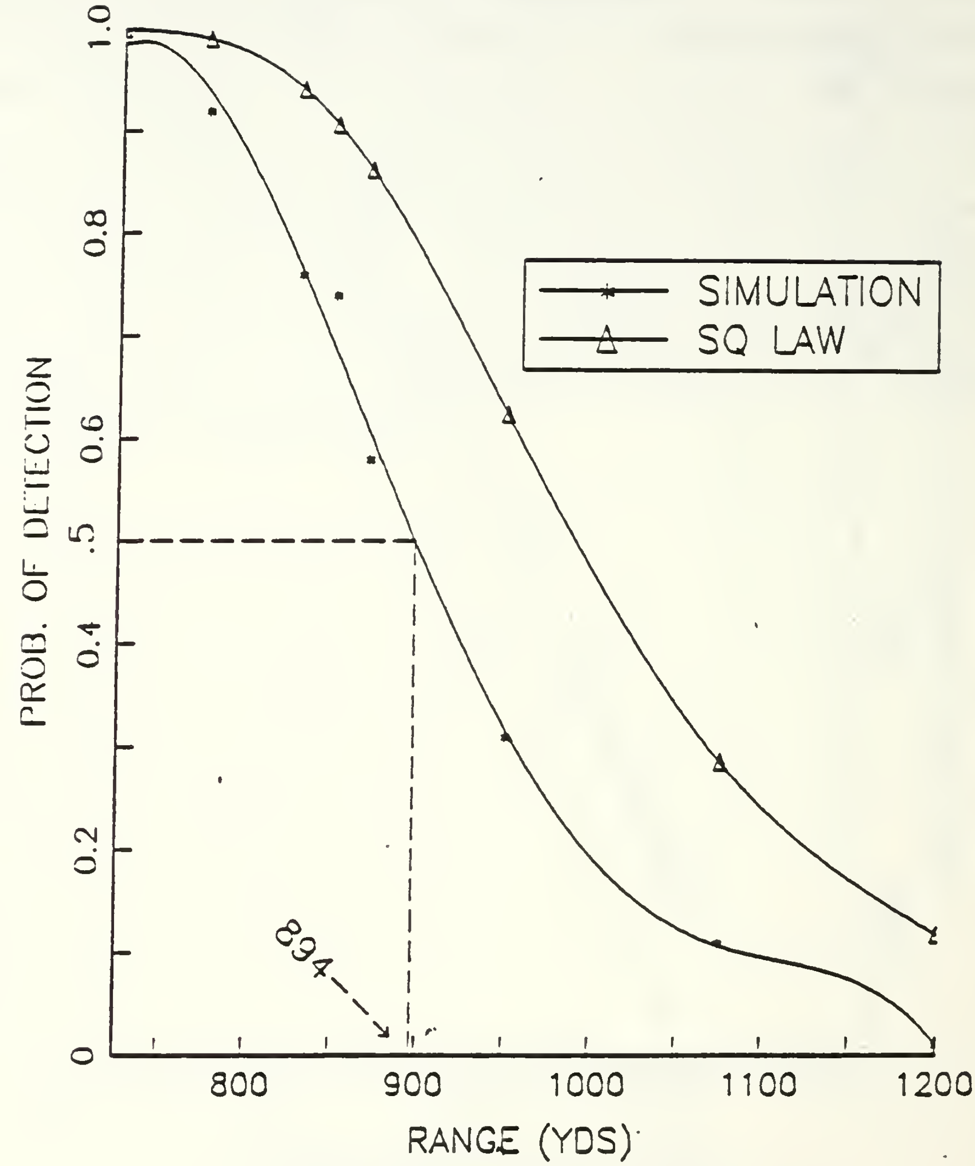

Figure 2.9: Lateral Range Curve
T=333 Seconds P_f=10^{-4}

TABLE 2.5: LATERAL RANGE CURVE DATA
T=333 SECONDS $P_f=10^{-4}$

LATERAL RANGE (YDS)	PROBABILITY OF DETECTION	
	SIMULATION	SQUARE LAW
625	1.00	1.0000
725	.99	.9993
775	.92	.9904
830	.76	.9414
850	.74	.9067
870	.58	.8625
950	.31	.6239
1075	.11	.2850
1200	.01	.1170

T=333 seconds $p_f = 10^{-5}$

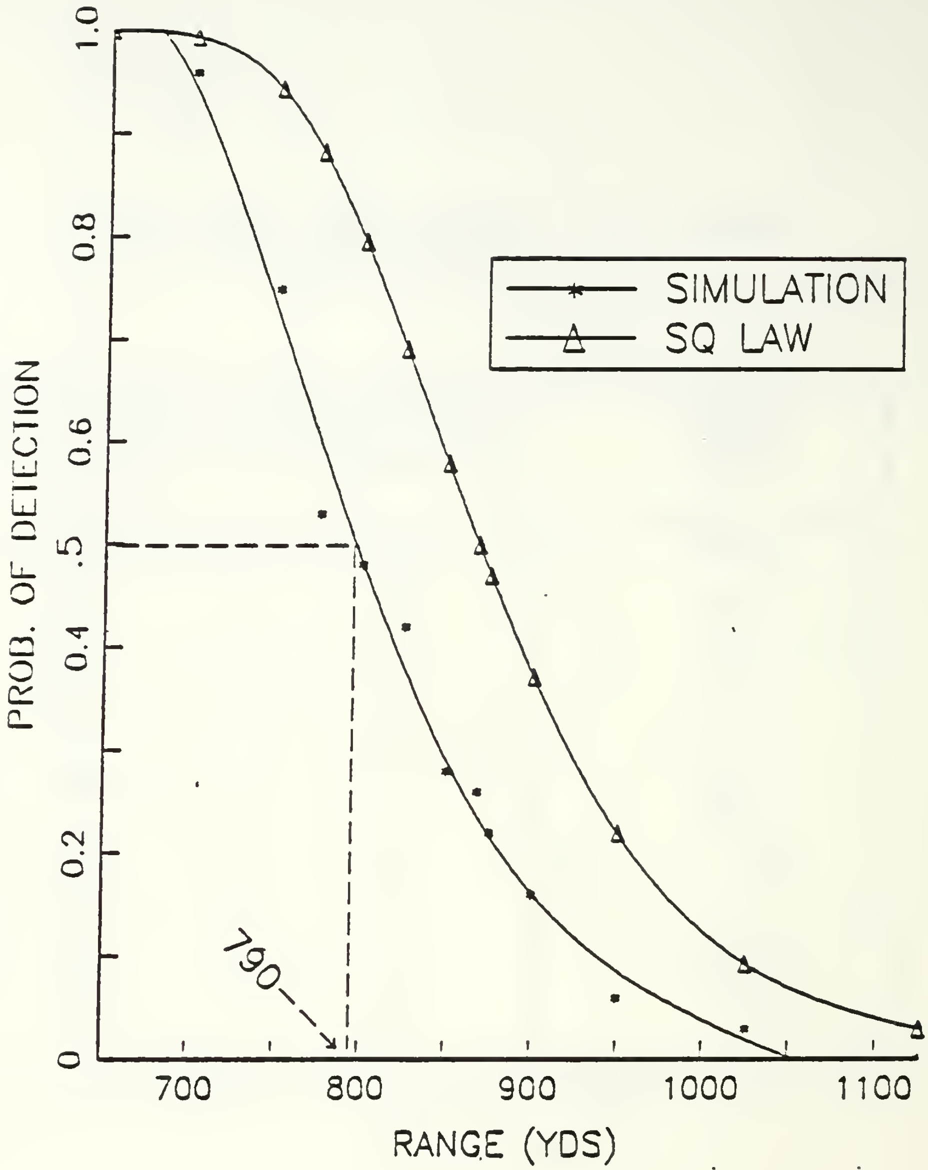

Figure 2.10: Lateral Range Curve
T=333 Seconds $P_f = 10^{-5}$

TABLE 2.6: LATERAL RANGE CURVE DATA
T=333 SECONDS $P_f = 10^{-8}$

LATERAL RANGE (YDS)	PROBABILITY OF DETECTION	
	SIMULATION	SQUARE LAW
720	.97	.9971
760	.87	.9793
780	.80	.9573
820	.68	.8750
830	.54	.8463
850	.46	.7808
880	.35	.6691
950	.22	.4090
1025	.08	.2122
1125	.00	.0826

T=312 seconds $p_f = 10^{-6}$

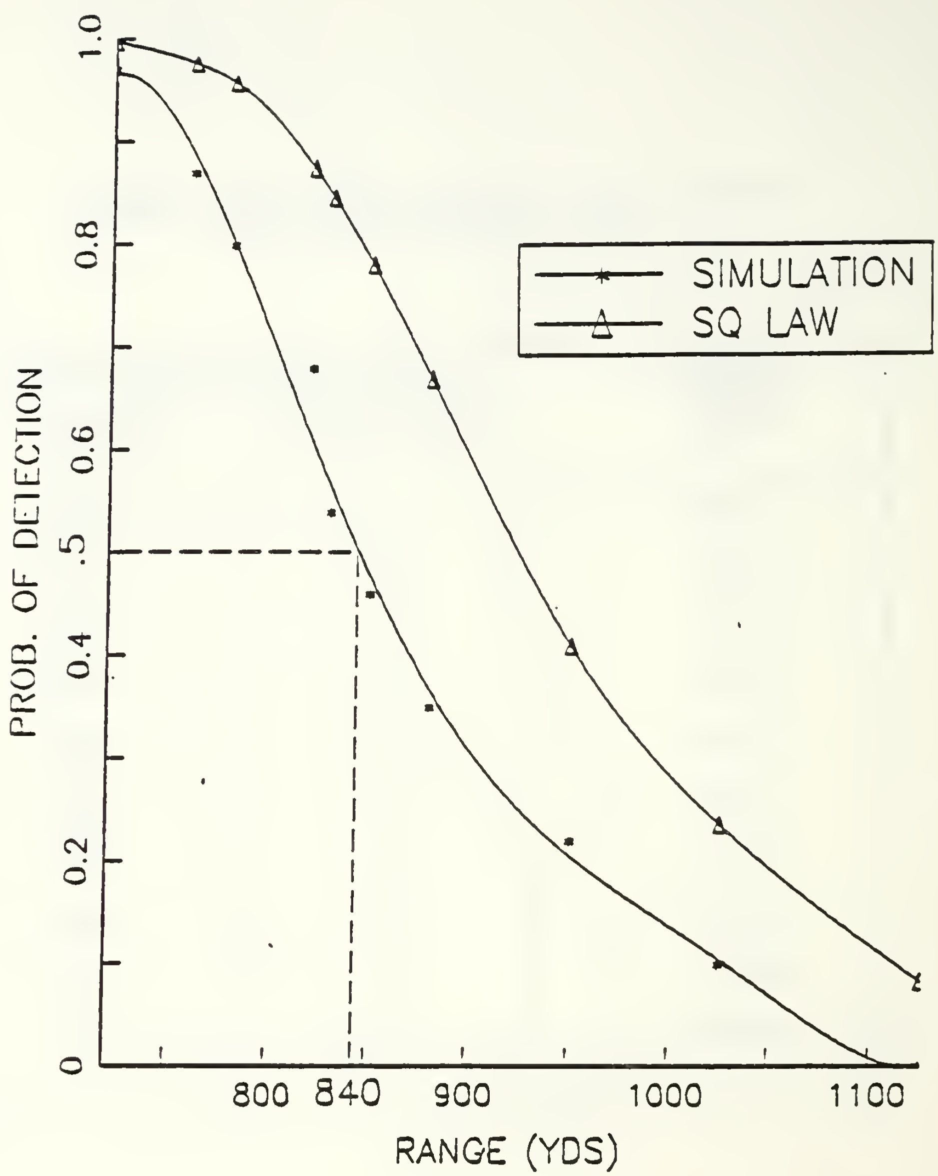

Figure 2.11: Lateral Range Curve
T=312 Seconds $P_f = 10^{-6}$

TABLE 2.7: LATERAL RANGE CURVE DATA
T=312 SECONDS $P_f = 10^{-6}$

LATERAL RANGE (YDS)	PROBABILITY OF DETECTION	
	SIMULATION	SQUARE LAW
650	1.00	.9999
700	.96	.9940
750	.75	.9435
775	.53	.8824
800	.48	.7957
825	.42	.6906
850	.28	.5783
868	.26	.4989
875	.22	.4692
900	.16	.3709
950	.06	.2190
1025	.03	.0924
1125	.00	.0290

An inspection of all the lateral range curves generated showed a fairly rapid rate of change of p_d versus lateral range. This would suggest that a step function approximation to the lateral range curves may be appropriate. This step function sets p_d=0 for lateral ranges greater than the point where p_d=.5 (i.e., MDR) and p_d=1 for ranges less than the MDR. Figure 2.12 compares the "Cookie Cutter"

approximation to the simulation results with T=365 seconds and $p_f = 10^{-4}$.

Figure 2.13 shows the overall trend in MDR as integration time increases for the simulation version of the Square Law model. Based on the decreasing slope of this graph, it would appear that the point of diminishing returns for MDR versus integration time is reached when T is equal to approximately 330 to 360 seconds.

D. VALIDATION OF RESULTS

Without suitable operational data for comparison, it is difficult to assess the accuracy of a simulation. In the cases examined in this thesis, the results appear to be consistent and have an intuitive appeal in that it is not unreasonable to expect lateral range estimates of the simulation Square Law model to be less than that predicted by the constant signal Square Law model. Rather than just using the S/N ratio at the closest point of approach in the encounter, the Square Law simulation model uses a signal that varies over the entire integration cycle. Since we have assumed a straight line encounter, the amount of signal reaching the hydrophone is less at the start and at the end of the integration period (i.e., the target is at a greater range) than at the midpoint of the integration cycle which coincides with CPA. A review of equation B.9 indicates that a lesser signal(S) would imply a decrease in p_d for a given lateral range.

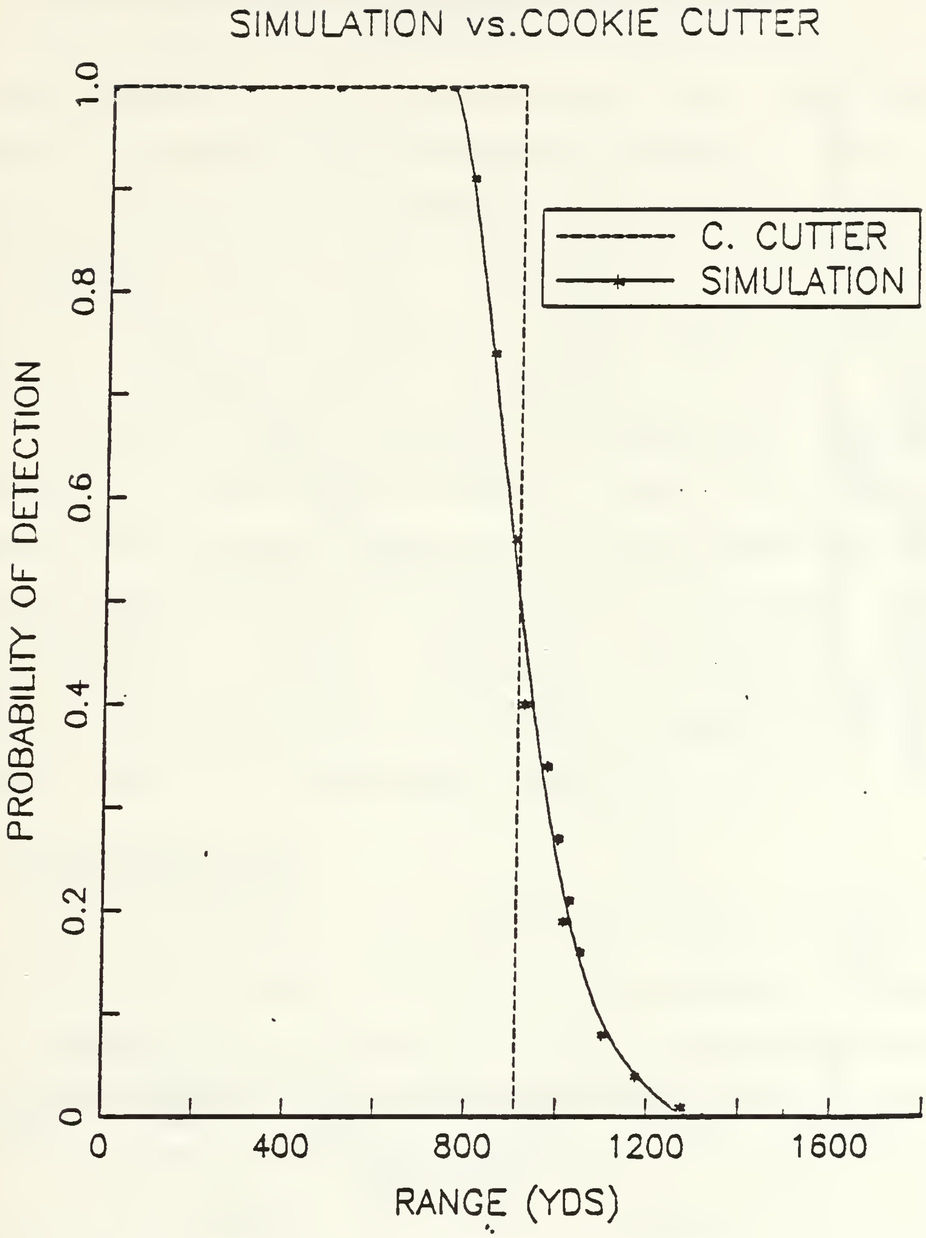

Figure 2.12: Cookie Cutter Model (MDR= 910 Yds) vs. Simulation Square Law Result T=365 Seconds P_f =10^{-4}

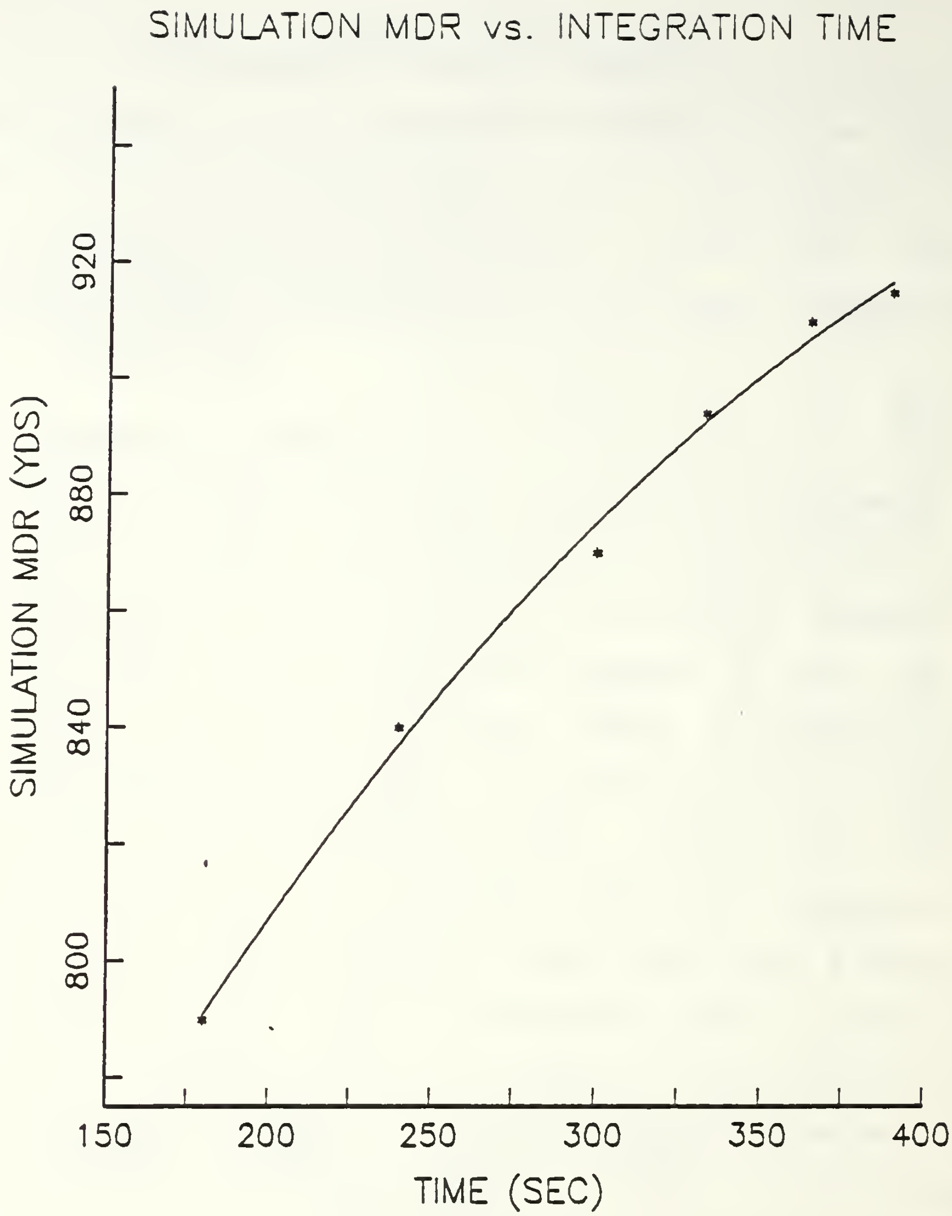

Figure 2.13:
MDR vs. Integration Time

III. TACTICAL IMPLICATIONS

The probability of a false alarm plays a key part in the success or failure of an auto-alert sonobuoy. If p_f is set too low, sensitivity decreases and detection ranges suffer. In this case either we run the risk of not detecting the target or we must employ a larger number of sonobuoys to achieve the same detection capability. If p_f is set too high, we run the risk of having to investigate numerous false contacts. In this case, we incur a "cost" in the form of the time and the sonobuoys required to investigate the alert. If the false alerts become so numerous that they can not be investigated on a real time basis, then the information may become tactically useless. Given its importance, how do we determine what is a reasonable value for p_f?

A. FALSE ALARM RATE

It is essential to understand that false alarm probability and false alarm rate are two distinct terms. False alarm probability is linked to the integration time of the detection system. It is the probability that the decision a target is present will be made when only "noise" is input to the detection system. This yes-no decision can be considered to be made once per integration period for an auto-alert sonobuoy. False alarm rate is defined here as the expected number of false alarms that will occur over a specified time interval. It is a function of integration time, the number of sensors employed, and the false alarm probability. All of these factors must be considered in order to achieve an

acceptable number of false alarms. That is, a number that can be investigated without having a detrimental impact on the tactical situation. For example, if the mission is to be a large area search and 400 sonobuoys are required, can the delivery platform meet search requirements plus carry enough buoys to transition to a tracking evolution for both valid contacts and false alarms?

B. EXPECTED VALUE APPROACH

The idea of having to maintain a "reserve" is a new issue. For the current inventory of sonobuoys, the number of sonobuoys required to deal with false alarms had a neglible impact on the total sonobuoy payload. In a large pattern (400-500 sonobuoys) auto-alert sonobuoy scenario, sonobuoy resource management would become a key issue.

The following example shows that part of the payload should be identified beforehand as a stockpile for investigating alerts. In the example, the following parameters are assumed:

1 VP aircraft

Sonobuoy search pattern = 400 sonobuoys

Total sonobuoy payload = 500 sonobuoys

Sonobuoy field monitor time = 3 hours

Sonobuoy integration time = 360 seconds

Sonobuoy tracking "reserves" = 50 sonobuoys

Sonobuoy alert "reserves" = 50 (for initial investigation).

In addition, it is assumed that one alert requires 25 sonobuoys and 30 minutes to investigate. The number of sono-

buoys required to investigate an alert is an estimate based on the author's operational experience.

By estimating the number of alerts that can be handled during a mission, we can apply an expected value approach in deriving p_f. In this example the maximum number of false alarms is

$$\frac{\#\text{ buoys avail. for alerts}}{\#\text{ buoys to invest. an alert}} = \frac{50}{25} = 2\ . \qquad \text{(eqn 3.1)}$$

Thus the acceptable false alarm rate is 2/3 per hour based on the three hour monitor period.

The number of decisions for a sonobuoy is a function of sonobuoy life and integration time with d_o the number of decisions,

$$d_o = \frac{\text{buoy life}}{\text{integration time}}\ . \qquad \text{(eqn 3.2)}$$

To determine p_f a binomial model is used. If X is the number of false alarms out of n decisions and X is a binomial random variable, then the expected number of false alarms for the n decisions is:

$$E[\ X\ ] = n \times p_f \qquad \text{(eqn 3.3)}$$

where

$$n = d_o \times \#\text{ of buoys.}$$

From equation (3.2)

d_o = 3 hrs / 360 sec = 10800/360 = 30 and,

solving for p_f yields:

$$p_f = \frac{E[X]}{n} = \frac{2}{30 \times 400} = 1.67 \times 10^{-4}\ . \qquad \text{(eqn 3.4)}$$

This value provides an informed basis to begin an evaluation of the system under consideration. In this scenario, the 50 "reserve" buoys would represent something other than an alert only sensor. Initial investigation of a contact would be done with auto-alert buoys. If a second alert is received in the area under investigation, the contact is assumed to be a valid contact and the 50 "reserve" sensors would be employed for identification and tracking. A merchant ship is a false contact that would trigger an alert. However, correlation via visual or electronic means should be sufficient to identify such targets prior to sensor deployment by an aircraft. To minimize the "cost" associated with investigating an alert, standard sonobuoys should have a communication channel allocation scheme that is separate from that of the auto-alert sonobuoys and that would reserve a block of channels for the tracking phase that is independent of the number of auto-alert sonobuoys in the search pattern.

C. OPTIMIZATION OF INTEGRATION TIME

Integration time determines the number of observations used to decide whether or not a target is present. If it is not properly matched to the signal duration, a degradation in recognition differential occurs. If the cycle is too long, more noise is added; if it is too short, signal is missed. This is discussed analytically in Appendix C for the Square Law model described in Appendix B.

A measure of effectiveness used in evaluating an acoustic detection system is median detection range (MDR) for the straight line encounter. In attempting to match signal duration and integration time, progress will be measured as

an improvement in MDR. In a straight line encounter with a sonobuoy, a target passes the sensor in a straight line at a constant speed. The plot of encounter closest point of approach versus encounter probability of detection yields a lateral range curve. By the definition used here, the CPA range at which the lateral range curve equals .5 is the MDR.

To estimate an optimum integration time, a straight line encounter was used with a single integration time centered at the CPA time. That is, the midpoint of the integration period occurs at the CPA.

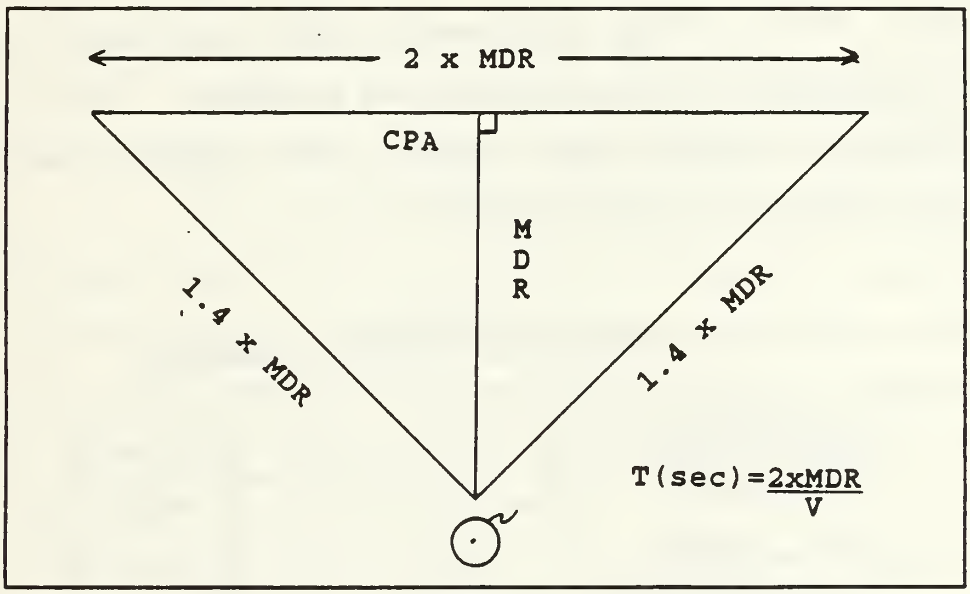

FIGURE 3.1
Optimum Signal Duration
in a Straight Line Encounter

Matching integration time to signal duration requires some thought. Analysis of some of the initial lateral range curves generated by the computer model (discussed in Chapter II) showed that the probability of detection for an encounter

approached p_f (i.e, p_d was close to zero) when the lateral range was approximately equal to 1.4 X MDR. This was interpreted as an indication that outside this range, little detectable signal was available. An application of the Pythagorean Theorem (Figure 3.1) made an integration time estimate equivalent to 2 X MDR divided by the assumed speed of the target a logical starting point.

C. CHOOSING ALTERNATIVES

The criticality of integration time as an input parameter is obvious. For a given ocean environment, however, it is the target characteristics that will drive the problem. A sensor with preset parameters must be able to perform satisfactorily in all envisioned scenarios. For the sake of discussion, assume three likely speed ranges for a hypothetical target as listed in Table 3.1.

TABLE 3.1
HYPOTHETICAL TARGET PARAMETERS

ALTERNATIVE	A_1	A_2	A_3
SPEED(KTS)	8-12	13-15	16-20
AVE. SPEED	10	14	18
SL(BAND LEVEL)	123	127	130

Detection system parameters and environmental data are listed in Table 3.2.

TABLE 3.2
HYPOTHETICAL SYSTEM PARAMETERS

BW = Receiver bandwidth = 100 hz
F1= Lower frequency = 800 hz
F2= Upper frequency = 900 hz
NL= Noise level = 63 dB (SPL) at 850 hz
DI= Directivity Index = 3 dB
pf= 1 $\times 10^{-4}$ (same order of magnitude as eqn. 3.4)
d = detection index ≈ 13.8 when p_d = .5
T = integration time (variable)

Suppose we define states of nature as:

S_i = { target with characteristics A_i appears }

where the events S_i are considered mutually exclusive and $\Sigma P(S_i) = 1$. The actual probabilities are unknown but could be estimated from historical data if one wished to treat this as a decision under conditions of risk. Using the procedure developed in the previous section for estimating a tactically optimum integration time (T^*), an estimate of median detection range (MDR) for each alternative (A_i) versus state of nature (Si) can be constructed. A revised RD is calculated using equation (C.1). A signal duration/integration time mismatch implies an increase in the S/N required to maintain the same p_d. Thus detection range decreases. The diagonal elements of the payoff matrix, Table 3.3 represent the best we could do with perfect information.

TABLE 3.3
PAYOFF MATRIX

PAYOFF MDR(YDS)	S_1	S_2	S_3
A_1=10 KTS T*=365 SEC	1013	1605	2267
A_2=14 KTS T*=430 SEC	972	1672	2361
A_3=18KTS T*=487 SEC	942	1620	2436

Decision making under conditions of uncertainty can now be employed [Ref. 3:p. 25-27]. The first criteria we apply is called the "Wald criteria". This is a conservative approach which gives the best guaranteed payoff. The inherent pessimistic nature of this procedure gives it wide military applicability where a worst case outcome could be disastrous and therefore must be avoided at all cost. It is also known as MAXIMIN (the minumum payoff for each alternative is calculated; the maximum of these is chosen). Alternative 1 is picked here. The next method is known as the Laplace or "equally likely" approach. Each state of nature is given the same chance of occurrence and an expected value for each alternative is calculated. Bigger is better and the highest payoff wins. In this case alternative 2 is the best choice with alternative 3 a close second. Another option, minimizing regret, suggests a "second-guess" approach. Here the decision maker looks back on the decision to see how much better the outcome would have been if the exact state of nature was known. The payoff for each alternative is then

subtracted from the best payoff in each column; the maximum for each row is observed. The smallest value from these has the least regret and becomes the basis for the decision.

TABLE 3.4
REGRET MATRIX

REGRET	S_1	S_2	S_3	MAX
A_1	0	67	169	169
A_2	41	0	75	75
A_3	71	52	0	71

Here, alternative 3 surfaces as the best course of action. Results from the three criteria are depicted in Table 3.5. It is interesting to note that a different alternative turned out to be the best choice for each criterion. One strategy is not the clear-cut winner and the decision maker must rely on knowledge and experience to make the call. By constructing a matrix of this sort however, the options have at least been exposed to different measures of effectiveness and possible trade-offs identified.

TABLE 3.5
SUMMARY OF DECISION CRITERIA

	MAXIMIN WALD PESSIMIST	LAPLACE EQUALLY LIKELY	MINIMAX LEAST REGRET
A_1	1013	1628	169
A_2	972	1668	75
A_3	942	1666	71
BEST	A_1	A_2	A_3

IV. Summary

The main thrust of this thesis was to estimate a broad-band auto-alert sensor's performance through an application of statistical detection theory. A simplified detection model based on a Square Law detector and a constant signal variance was compared to a computer simulation that incorporated a Square Law detector in which signal variance is not constant. In particular an auto-alert sensor with an integration time of approximately 360 seconds and p_f equal to 10^{-4} was examined. In all instances the computer simulation model yielded lateral range curves that were below those for the simplified model. Median detection range estimates for the simulation model were approximately 10 percent less than those for the simplified model. The steep slope of the generated lateral range curves at the MDR indicated that both models could be described as "Cookie Cutter" detection models.

The importance of establishing a false alarm rate that meets tactical requirements while remaining within sonobuoy payload limitations was discussed. An expected value approach for determining the number of false alerts that could be reasonably investigated during a mission was examined as a means of establishing a realistic p_f. Exposing design criteria to evaluation under differing states of nature (target scenarios) and multiple measures of effectiveness was offered as a decision making tool. Using

an estimate for integration time equal to (2 x MDR)/velocity appeared to yield an MDR that was "tactically optimum".

In closing, it might be well to consider an insight offered by Urick [Ref. 4:p. 28]:

> A limitation of another kind is produced by the nature of the medium in which sonars operate. The sea is a moving medium containing inhomogenities of various kinds, together with irregular boundaries, one of which is in motion. Multipath propagation is the rule. As a result, many of the sonar parameters fluctuate with time, while others change because of unknown changes in the equipment and the platform on which it is mounted. Because of these fluctuations, a "solution" of the sonar equations is no more than a best-guess time average of what is to be expected in a basically stochastic problem.

APPENDIX A: THE SONAR EQUATION

The desire to tactically exploit SONAR (Sound Navigation and Ranging) during World War II led to the formulation of the active and passive sonar equations. These relationships still remain as one of the primary analysis tools for studying and predicting SONAR performance.

The physical relationships that govern the terms in the SONAR equations are discussed in most basic acoustic texts. Only the passive SONAR equation is of interest in this thesis.

In using the passive sonar equation to predict system performance, it is useful to write it in the form

$$10 \times \log(S/N) = SL - TL - (NL - DI) \qquad \text{(eqn A.1)}$$

where:

TL = transmission loss

SL = source level of the target

NL = noise level

DI = directivity index

The unit of measure for the terms in the sonar equation is the decibel (dB). The unit of reference in this thesis is one micropascal (μpa).

Source levels are referenced as eminating from a point source one yard from the actual source. In actuality, they are measured at a greater distance and extrapolated back towards the source based on a spherical spreading loss. Transmission loss is the energy loss that occurs as the sound travels from the target to the hydrophone. When dealing with

relatively short ranges and frequencies below 10 khz, it is permissible to approximate transmission loss as:

$$TL = 20 \times \log r \qquad \text{(eqn A.2)}$$

where r is the range in yards from the source to the detector. Noise level generally encompasses all "ambient" noises generated in the ocean that are not part of the desired signal. Merchant shipping, biologics, wind, and waves all combine in establishing this number. Tables are available for estimating deep-water ambient noise as a function of frequency [Ref. 4:p. 189]. Directivity index is a measure of the ability of the receiver to discriminate against noise arriving from a direction other than than that of the target. It reduces the overall noise level measured at the hydrophone. If the noise or signal are measured over a one hertz band, the term spectrum level (SPL) is used. When summing over the entire bandwidth (W) of the receiver, the correct terminology is band level (BL). The relationship is:

$$BL = SPL + 10 \times \log (W) \; . \qquad \text{(eqn. A.3)}$$

Caution must be exercised when computing a band level over a large frequency range that does not have a constant spectrum level. The region must be subdivided into smaller intervals with relatively constant spectrum levels. The resulting band levels for each interval are then added using a logrithmic power sum to obtain the overall band level. A nomograph offered by Kinsler et al [Ref. 5:p. 246] simplifies this procedure.

APPENDIX B: STATISTICAL DETECTION THEORY

In the early 1950's Peterson, Birdsall and Fox [Ref. 6] applied statistical methods to the evaluation of detection system performance. These methods are most applicable to systems such as an auto-alert sonobuoy that do not involve an operator.

The ability to detect a signal in a background of noise implies that a decision must be made. Using an approach similar to Forrest [Ref. 7:pp. 1-6], we define the events:

H_0 = { noise alone is present }

H_1 = { signal and noise are present }

D_0 = { detector decides input is noise alone }

D_1 = { detector decides input is desired signal }.

The Venn diagram of figure B.1 summarizes these events.

	H_0	H_1
D_0	$D_0 \cap H_0$ CORRECT "NO" DECISION	$D_0 \cap H_1$ INCORRECT "NO" DECISION MISSED DETECTION
D_1	$D_1 \cap H_0$ INCORRECT "YES" DECISION FALSE ALARM	$D_1 \cap H_1$ CORRECT "YES" DECISION VALID TARGET

Figure B.1: Possible Detection Events

The probability of a false alarm and the probability of detection can be defined as the conditional events:

$$p_f = P(FA) = P(\, D_1 \mid H_0 \,) = \alpha \qquad \text{(eqn B.1)}$$

$$p_d = P(DET) = P(\, D_1 \mid H_1 \,) = 1-\beta \, . \qquad \text{(eqn B.2)}$$

Note that in statistical hypothesis testing, p_f is analagous to α, the probability of a Type I error (rejecting a true hypothesis); and 1-pd is analagous to β, the probability of a Type II error (accepting a false hypothesis) [Ref. 8:pp. 315-316]. If we define Y (a vector) as the input to a receiver (the decision portion of a detection system) and the conditional probability density functions of Y are known, i.e., $f_y(y|H_0)$ for noise alone and $f_y(y|H_1)$ for signal and noise, a likelihood ratio approach can be pursued in defining the event D_1. The likelihood ratio is

$$L(y) = \frac{f_y(y|H_1)}{f_y(y|H_0)} \, . \qquad \text{(eqn B.3)}$$

L(y) can be viewed as the conditional odds favoring the occurrence of the event $H_1=\{$ signal and noise are present $\}$. If the event $D_1=\{ Y \varepsilon R \}$, where $R=\{ y: L(y) \geq K\}$, then D_1 is determined by a Neyman-Pearson criterion [Ref. 9:p. 419]. This implies that the probability of detection(p_d) will be a maximum for a given false alarm probability(p_f). By defining the events:

$H_0: y_i = n_i$ with n_i $N(0, \sigma^2)$

$H_1: y_i = n_i + s_i$ with s_i $N(0, \sigma^2_s)$

and treating the n_i and the s_i as independent normal random variables, the distribution for the combined signal and noise is $N(0, \sigma^2+\sigma^2_s)$. [Note: the notation $N(0, \sigma^2)$ indicates a normal distribution with a mean = 0 and the variance = σ^2]. Applying a likelihood ratio criterion as in equation (B.3) yields

$$X = \sum_{i=1}^{m} Y_i^2$$

as a test statistic. When H_0 is true,

$$X/\sigma^2 = \sum_{i=1}^{m} (Y_i/\sigma)^2$$ has a chi-square distribution with m degrees of freedom. When H_1 is true,

$$X/(\sigma^2+\sigma^2_s) = \sum_{i=1}^{m} (Y_i^2/(\sigma^2+\sigma_s^2))$$ also has a chi-square distribution. As m, the number of observations becomes large (m>31), the chi-square distribution (μ=m, σ^2=2m) can be approximated by a normal distribution [Ref. 9:p. 292]. With this approximation,

$$p_f = 1 - \Phi[\ \{(X^*/N)-m\} / \sqrt{2m}\ ] \qquad \text{(eqn B.4)}$$

where $\Phi(Z)$ is the standard normal cumulative distribution function. Setting $v^* = \{\ (X^*/N)-m\ \} / \sqrt{2m}\ \}$, equation B.4 becomes

$$p_f = 1 - \Phi[\ v^*\ ]. \qquad \text{(eqn B.5)}$$

And, in addition,

$$p_d = 1 - \Phi\left\{ \frac{1}{1 + S/N} \left[v^* - \left[\frac{m}{2}\left(\frac{S}{N}\right)^2 \right]^{1/2} \right] \right\} \qquad \text{(eqn B.6)}$$

where

$$N = \text{Noise power} = E[\ 1/m \sum_{i=1}^{m} n_i^2\ ] = \sigma^2 \quad \text{and}$$

$$S = \text{Signal power} = E[\ 1/m \sum_{i=1}^{m} s_i^2\ ] = \sigma_s^2.$$

Based on the stochastic sampling theorem [Ref. 10:p. 370]

$$\Delta t = 1/(\ 2 \times BW\ ) \quad \text{and} \qquad \text{(eqn. B.7)}$$

$$m = T/\Delta t = 2T \times BW \qquad \text{(eqn. B.8)}$$

where

m = # of observations

T = integration time

Δt = sampling interval

BW = frequency bandwidth.

With the value of m replaced by 2T(BW), p_d can be written as

$$p_d = 1 - \Phi\left[\frac{1}{1+ S/N} (v^{*} - d^{½}) \right] \qquad \text{(eqn B.9)}$$

where $d = t \times BW \times (S/N)^2$. The previous development parallels the approach used by Forrest [Ref. 7:p. 10].

Figure B.2 illustrates the relationship between the probability distributions for the case when S/N << 1. The quantity d is referred to as the "detection index"; $d^{½}$ is the separation of the means in this case.

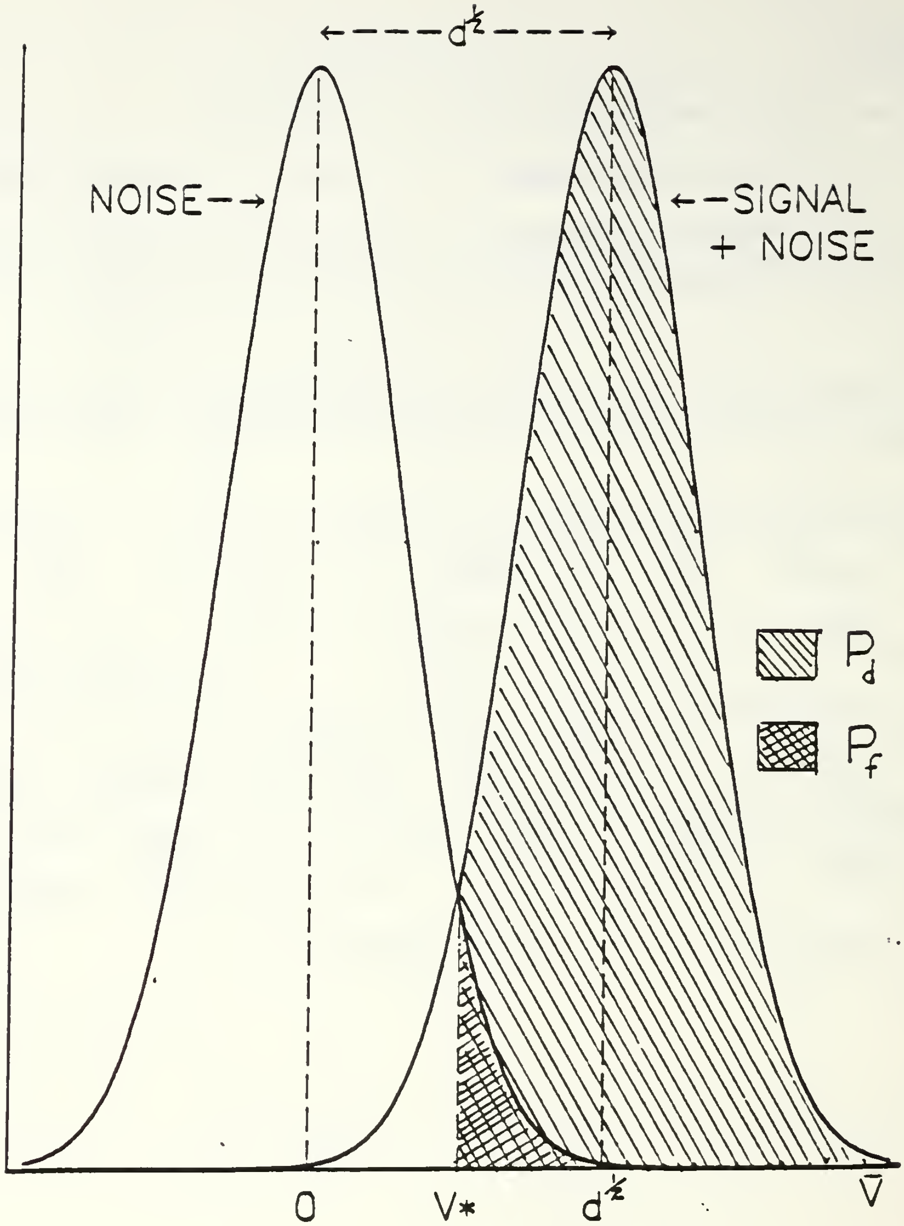

Figure B.2: Gaussian Distributuions for the Random Variable V for Noise and Signal + Noise

For a given threshold setting(v^*) and detection index(d), a family of curves relating p_f and p_d can be constructed. These are called Receiver Operating Characteristc (ROC) curves. Figure B.3 provides an example for the case S/N << 1.

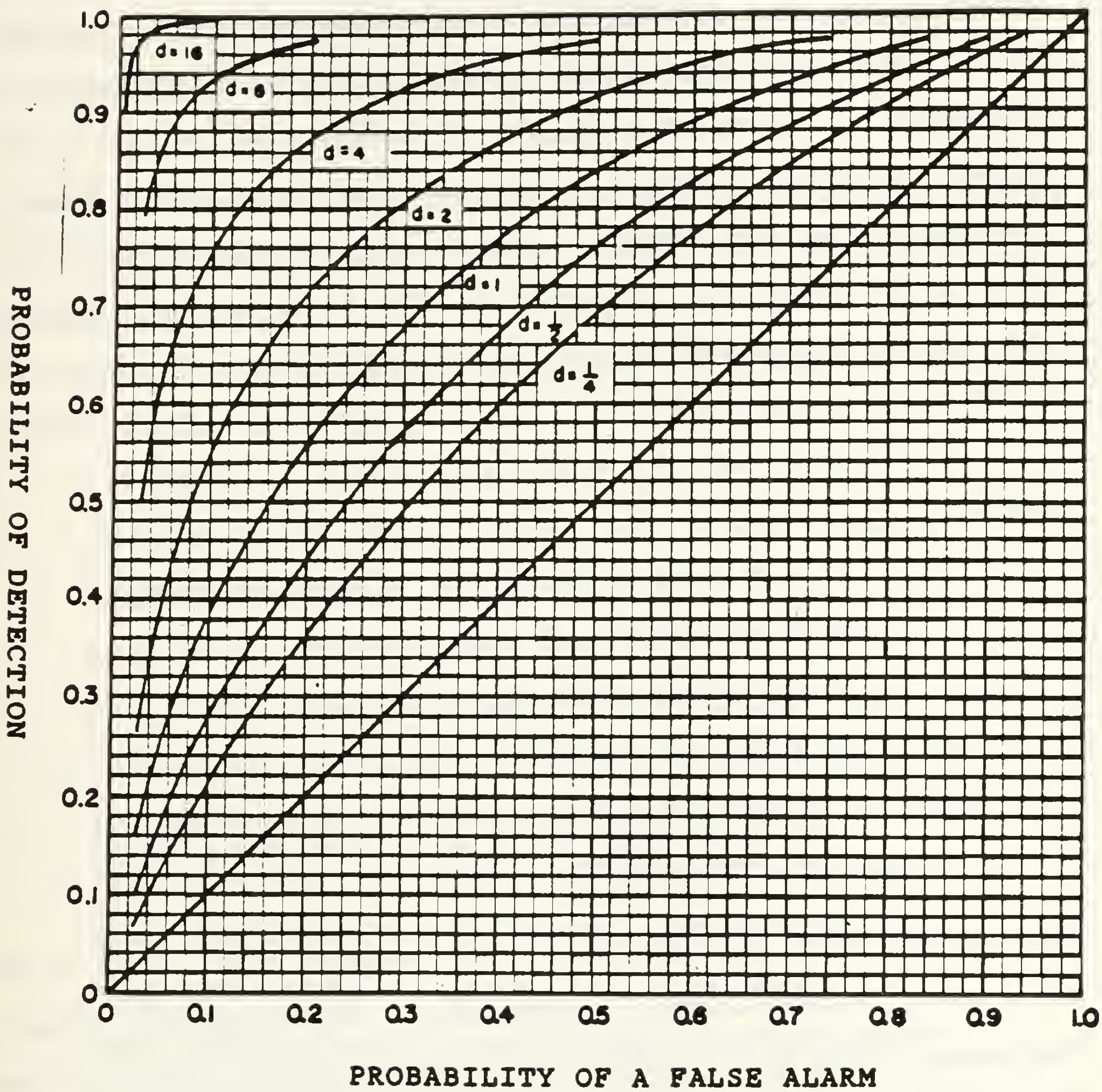

Figure B.3: Receiver Operating Characteristic (ROC) Curve

APPENDIX C: MATCHING INTEGRATION TIME TO SIGNAL DURATION

A detailed explanation for matching signal duration to integration time is offered by Bartberger [Ref. 11:pp. 365-367]. Once p_f has been established, the Square Law model discussed in Appendix B allows an analytical solution for the signal-to-noise ratio required to achieve a specified probability of detection. When p_d is equal to .5, this detection threshold is known as the recognition differential and can be expressed as

$$RD = 10 \times \log(S/N)^*$$
$$= 5 \log[d/(BW\ t)] + |5 \log(T/t)| \qquad \text{(eqn C.1)}$$

where:

$d = T \times BW \times (S/N)^2 =$ detection index

$BW =$ bandwidth of the receiver

$T =$ integration time (seconds)

$t =$ signal duration (seconds)

$(S/N)^* =$ signal to noise ratio for a probability of detection of .5 at a specified level of performance.

The second term in equation (C.1) equals zero when integration time equals signal duration. The absolute value sign indicates that the recognition differential always increases when a mismatch occurs.

APPENDIX D: PROGRAM LISTING

```
* PROGRAM: CALCULATES P(DETECTION) USING SIMULATION FOR A
* SQUARE LAW MODEL WHEN THE CONSTANT SIGNAL VARIANCE
* ASSUMPTION IS RELAXED.  UPDATED 27 FEB 88.

* CHECK ALL PARAMETER AND DATA STATEMENTS TO ENSURE PROPER
* PARAMETERS  HAVE BEEN ENTERED. "NUM" MUST BE CALCULATED
* AHEAD OF TIME.
* NUM=INT.TIME/SAMP.RATE/# OF SUBDIV. = # SAMPLES PER SUBDIV.
* SAMP. RATE = 1/(2*BW) = DT

      INTEGER TRIAL, REPS, VALUE, DF, SSEED, SUBDIV
      INTEGER SAMPLE
      REAL MOVE, NOISE, INPUT,NBW
      PARAMETER (FREQ1=800,FREQ2=900, IT=365, PFA=.0001)
      PARAMETER (NL1=63,DI=3, NUM=36500,REPS=100, SUBDIV=2)
      PARAMETER (SL1=103,SPD=10,VALUE=3)
      REAL NRV(NUM),SRV(NUM), PDET(VALUE), DETRGE(VALUE)
      REAL SQLPD(VALUE), SI(VALUE), SNR(VALUE)
      REAL DEE(VALUE), ZEE(VALUE), PP(VALUE)
      REAL AVESIG(VALUE),AVEPD(VALUE),AVSDB(VALUE)
      DATA ISORT,MUL,IX1,IX2/0,1,88324,72617/
      DATA (DETRGE(I),I=1,32)/1000,900,1350/

* REMEMBER RANDOM SEED VALUES

      NSEED=IX1
      SSEED=IX2
```

```
* CALCULATE INVERSE NORMAL CDF FOR DESIRED PROB(FALSE ALARM).
* VALID IF DEGREES OF FREEDOM(DF) > 30.

      P=1-PFA
      CALL MDNRIS(P,Z,IER)

* CALCULATE PARAMETERS AND NORMAL APPROX FOR CHISQUARE
* CRITICAL VALUE.

      D=Z**2
      BW=FREQ2-FREQ1
      DT=1/(2*BW)
      DF=IT/DT
      SBL=SL1+10*(LOG10(BW))
      NBW=10*(LOG10(BW))
      DNL=NL1-DI+NBW
      VARN=10**(DNL/10)
      SIGMA=VARN**.5
      RD=5*(LOG10(D/(IT*BW)))
      CHISQ=DF+(Z*((2*DF)**.5))
      XCRIT=CHISQ*VARN
      VSTAR=(CHISQ-DF)/((2*DF)**.5)

* CALCULATE  CONVERSION FACTOR=YARDS TRAVELLED PER SECOND PER
* KNOT. MOVE IS THE DISTANCE TRAVELLED IN YDS DURING TIME DT.

      CF=2000/3600
      MOVE=CF*SPD*DT
```

```
* SETUP LOOP FOR EACH DETECTION RANGE VALUE: INITIALIZE
* DETECTION COUNTER. CPA IS THE RANGE AT CLOSEST POINT OF
* APPROACH.

      DO 5 K=1,VALUE
         DET=0
         CPA=DETRGE(K)
         SIGSUM=0

* SETUP LOOP FOR REQUIRED REPLICATIONS. INITIALIZE SIGNAL +
* NOISE COUNTER FOR EACH INTEGRATION PERIOD.

      DO 10 TRIAL=1,REPS
         OUTPUT=0

* INITIALIZE TGT POSITION. XPOS IS THE STRAIGHT LINE DISTANCE
* FROM THE CPA POINT. MINUS SIGN INDICATES TIME PRIOR TO CPA.
* SIMULATES A STRAIGHT LINE ENCOUNTER SITUATION.  INTEGRATION
* TIME IS A "WINDOW" CENTERED ON THE CPA.

      XPOS=(CF*SPD*IT)*(-.5)

* SUBDIVIDE INTEGRATION CYCLE INTO INCREMENTS TO AVOID
* COMPUTER MEMORY PROBLEMS (APPROX 200K IN ARRAYS IS THE
* LIMITING POINT).

         DO 15 I=1,SUBDIV

* GENERATE N(0,1) DISTRIBUTION THAT WILL BE USED TO TRANSFORM
```

```
* SIGNAL PLUS NOISE INTO A GAUSSIAN SUM.  NOTE: NOISE AND
* SIGNAL VARY IN A "WHITE NOISE" FASHION.

          CALL LNORM(IX1,NRV,NUM,MUL,ISORT)
          CALL LNORM(IX2,SRV,NUM,MUL,ISORT)

* CALCULATE TARGET RANGE FROM BUOY. RANGE IS THE HYPOTENUSE
* OF A RIGHT TRIANGLE FORMED BY XPOS AND CPA VECTORS.
* RANDOMIZE NOISE COMPONENT AND TRANSFORM TO N(0, VARN).
* RANDOMIZE SIGNAL COMPONENT AS A FUNCTION OF RANGE;
* TRANSFORM  TO N(0, VARSIG). SUM S+N AND SQUARE. SUM SQUARED
* INPUT OVER INTEGRATION CYLE IN TIME STEPS DT.
           DO 20 SAMPLE=1, NUM
              RANGE=((XPOS*XPOS)+(CPA*CPA))**.5
              SIR=SBL-(20*LOG10(RANGE))
              COEFF=SIR/10
              SIGVAR=10**COEFF
              SIGDEV=SIGVAR**.5
              NOISE=NRV(SAMPLE)*SIGMA
              SIGNAL=SRV(SAMPLE)*SIGDEV
              SIGSUM=SIGSUM + (SIGNAL)**2
              INPUT=NOISE+SIGNAL
              SQUARE=INPUT*INPUT
              OUTPUT=OUTPUT+SQUARE
              XPOS=XPOS+MOVE
 20        CONTINUE
 15        CONTINUE

* DECIDE IF DETECTION HAS OCCURRED AND INCREMENT COUNTER.
```

```
* NOTE THAT ONLY ONE 'DECISION' IS MADE PER COMPLETE
* INTEGRATION CYCLE. (XCRIT IS X* IN THEORY SECTION.)

          IF (OUTPUT .GE. XCRIT) THEN
             DET=DET+1
          ENDIF
 10      CONTINUE
* CALCULATE PROBABILITY OF DETECTION AND AVERAGE SIGNAL
* PRESENT DURING THE ENCOUNTER.  TRANSFORM SIGNAL INTO AN
* AVERAGE LEVEL IN DECIBELS.

      PDET(K)=DET/REPS
      AVESIG(K)=SIGSUM/(NUM*REPS*SUBDIV)
      AVSDB=10*(LOG10(AVESIG(K)))

 5     CONTINUE

* PRINT OUT PARAMETERS AND RESULTS.

       PRINT*
       PRINT 24,'P(FALSE ALARM) =',PFA
 24    FORMAT(1X,A16,1X,F9.8)
       PRINT*,'RECOGNITION DIFFERENTIAL= ',RD
       PRINT*,'INTEGRATION TIME= ',IT
       PRINT*,'BANDWIDTH =',NINT(BW)
       PRINT*,'DETECTION INDEX =',D
       PRINT*,'SOURCE LEVEL= ',NINT(SBL)
       PRINT*,'DETECTED NOISE LEVEL= ',NINT(DNL)
       PRINT*
```

```
      PRINT*,'CRITICAL VALUE= ',XCRIT
      PRINT*,'CHISQUARE= ',CHISQ
      PRINT*,'DEGREES OF FREEDOM= ',DF
      PRINT*,'VSTAR= ',VSTAR
    PRINT*
      PRINT 25,'RANGE','P(DETECT)'
 25   FORMAT(3X,A5,T15,A9 /1X,25('='))
      DO 30 L=1,VALUE
         PRINT 35,DETRGE(L),PDET(L)
 35      FORMAT (3X,F5.0,T16,F6.4)
 30   CONTINUE

* CALCULATE SQUARE LAW P(DETECT)

      PRINT 40,'RANGE','SQLAW P(DETECT)','DET. INDEX'
 40   FORMAT(/3X,A5,T15,A16,T34,A10 / 1X,45('='))
         DO 50 M=1,VALUE
            SI(M)=SBL-(20*LOG10(DETRGE(M)))
            SNR(M)=(10**(SI(M)/10))/VARN
            DEE(M)=(DF/2)*(AVESIG(N)/VARN)**2
            ZEE(M)=VSTAR-(DEE(M))**.5
            CALL MDNOR(ZEE(M),PP(M))
            PRINT 45,DETRGE(M),SQLPD(M),DEE(M)
 45         FORMAT(3X,F5.0,T21,F6.4,T38,F5.2)
 50      CONTINUE

*CALCULATE P(DETECT) BASED ON "AVE" SIGNAL INTENSITY.

     PRINT 55,'RANGE','AVESIG P(DET)','AVE SIG INTENSITY'
 55   FORMAT(/3X,A5,T15,A13,T32,A17 / 1X,50('='))
```

```
      DO 60 N=1,VALUE
         DEE(N)=(DF/2)*(AVESIG(N)/VARN)**2
         ZEE(N)=VSTAR-(DEE(N))**.5
         CALL MDNOR(ZEE(N),PP(N))
         AVEPD(N)=1-PP(N)
         PRINT 65,DETRGE(N),AVEPD(N),AVESD(N)
65       FORMAT(3X,F5.0,T21,F6.4,T38,F6.3)
60    CONTINUE
*PRINT OUT RANDOM SEED VALUES.
      PRINT*
      PRINT*,'NOISE RANDOM SEED= ',IX1
      PRINT*,'SIGNAL RANDOM SEED=',IX2
      END
```

APPENDIX E: KEY VARIABLES

Key variables are listed in the order in which they appear in the FORTRAN program.

NSEED = random number seed for noise values

SSEED = random number seed for signal values

PFA = probability of a false alarm

D = detection index

BW = bandwidth

DT = delta t, time interval between samples

IT = integration time (sec)

SBL = source level over the entire bandwidth

NBW = noise level over the entire bandwidth

DNL = detected noise level

NL1 = noise level in a 1 hz band

SL1 = source level in a 1 hz band

DI = directivity index

VARN = variance of the noise level

SIGMA = standard deviation of the noise level

CHISQ = chi-square value for the given parameters

XCRIT = X* = the threshold value

VSTAR = entering argument for eqn B.5 and B.9

CF = conversion factor (yds/sec/knot)

MOVE = distance target travels in one time increment

SPD = target speed in knots

VALUE = number of data points per program run

CPA = closest point of approach between the target and the buoy

SIGSUM = sum of the randomized signal values

DETRGE = detection range

OUTPUT = sum of the energy accumulated

XPOS = x position = target range from CPA in the straight line encounter scenario

SUBDIV = # of subdivisions in the integration cycle

NUM = the number of samples in one subdivision

SIR = signal intensity as a function of range

SIGVAR = variance of the signal distribution

SIGDEV = standard deviation of the signal distribution

NOISE = randomized value of the noise distribution

SIGNAL = randomized value of the signal distribution

INPUT = signal plus noise input to the hydrophone

SQUARE = the square of the input value

PDET = the probability of detection

AVESIG = average value of the randomized signal

AVSDB = average signal in decibels

SI = signal intensity at CPA

SNR = signal to noise ratio

DEE = detection index

A brief description of the subroutines used is provided for added clarity.

LNORM : provides streams of standard normal random variables using LLRANDOMII package.

MDNRIS : calculates the inverse of the standard normal distribution for a given probability.

MDNOR : calculates the cumulative distribution function for the standard normal distribution.

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Thesis
F4272 Filanowicz
c.1 An analysis of an
auto-alert sonobuoy
detection model.